Workshop in Convexity and Geometric aspects of Harmonic Analysis: Problem List, Group lists and photos of the discussions

December 2019

1 Problems

Problem 1 (Artstein-Avidan). If there exists $\delta > 0$ such that $K = t(\delta) K_\delta$, is it true that K is an ellipsoid? Here and below K_δ stands for the floating body. (this is known for L_p balls).

Problem 2 (Artstein-Avidan). Does there exist a convex body K such that $K = L_\delta$ and $K = M_\delta$, but $L \neq M$?

Problem 3 (Koldobsky). Let K and L be convex bodies containing a ball of radius $t > 0$. Assume that for every hyperplane H tangent to tB_2^n, we have that

$$|K \cap H|_{n-1} = |L \cap H|_{n-1}.$$

Does it imply that $|K| = |L|$? Or $K = L$? (known for $n = 2$ when K is a ball; also known for polytopes due to Yaskin.)

Problem 4 (Koldobsky). Suppose that for convex bodies $K, L \subset \mathbb{R}^n$, and for every $u \in S^{n-1}$, we have

$$\left|\partial\left(K \cap u^\perp\right)\right|_{n-2} \leq \left|\partial\left(L \cap u^\perp\right)\right|_{n-2}.$$

Does this imply that

$$|\partial K|_{n-1} \leq |\partial L|_{n-1}?$$

The answer is **no** for $n \geq 14$, Koldobsky-Konig.

Problem 5 (Artstein-Avidan). Fix $\delta > 0$. Find convex bodies K and L, for which the following extrema are attained

$$\max_{|K|=1} |K_\delta|, \qquad \min_{|L|=1} |L_\delta|.$$

Problem 6 (Artstein-Avidan). Can we prove the following Blaschke-Santalo type inequality for floating bodies:

$$|K_{\delta|K|}| \cdot |(K)^\circ_{\delta|K^\circ|}| \leq |(B_2^n)_{\delta|B_2^n|}|^2?$$

Problem 7 (Werner). Define Gaussian affine surface area as

$$asg\,(K) = \int\limits_{\partial K} \kappa^{\frac{1}{n+1}} \frac{1}{(2\pi)^n} e^{-\frac{|x|^2}{2}} d\mu_K(x).$$

For what K is $asg\,(K)$ maximized?

Problem 8 (Livshyts). Let γ be an isotropic log-concave probability measure with density e^{-v}. Define

$$\Gamma(\gamma) = \max_{K\text{- convex in}\,\mathbb{R}^n} \left\{ \int\limits_{\partial K} e^{-v} d\mathcal{H}^{n-1}(x) \right\}.$$

Show that $\Gamma(\gamma) \leq Cn$, where C is a positive absolute constant. (it is known that $\Gamma(\gamma) \leq Cn^2$, Livshyts; it is also known in the case of uniform distributions, Ball.)

Problem 9 (Artstein-Avidan). Take K to be a convex body of volume 1 and $\mathcal{E}$ to be an ellipsoid. Is

$$\arg\max_{|\mathcal{E}|=1} |K \cap \mathcal{E}|$$

unique? (would follow from the equality cases in the B-conjecture in case proven.)

Problem 10 (Alfonseca). Let K be an origin-symmetric convex body, and E be a k-dimensional subspace of $\mathbb{R}^n$. Is it true that if

$$|K|E| \cdot |P_{E^\perp} K| = \text{const},$$

then K is a ball? (a generalization of the 5th Busemann-Petty problem)

Problem 11 (Vempala). Let K be a convex body and $\mathcal{E}$ is an ellipsoid such that

$$|K \cap \mathcal{E}| \geq \frac{|\mathcal{E}|}{2}$$

Find a minimal α such that $|\alpha\mathcal{E} \cap K| \geq \frac{|K|}{2}$. Conjecture: $\alpha \approx \log n$.

Problem 12 (Livshyts).

a) Read Lee-Vempala paper;

b) Estimate the ratio of the Poincare constant and the inradius of an isotropic convex body and get a better answer than $Cn^{1/4}$ (addendum – equivalent to the KLS conjecture and not weaker, argument by Paouris).

Problem 13 (Tikhomirov).

a) Prove the non-asymptotic Marchenko-Pastur law.

b) Read paper of Nguyen (better probabilities using Nguyen's restricted invertibility property)

Problem 14 (Tikhomirov). Let $X_1, \ldots, X_n$ are random vectors and

$$H = span\,\{X_1, \ldots, X_{n-1}\},$$

where coordinates of X_i are i.i.d. random variables. Fix vector θ, find the optimal upper bound

$$\mathbb{P}\Big(\,\langle n_H, \theta\rangle \leq t\Big) \leq ?,$$

and characterize θ?

Problem 15 (Tikhomirov). Let $K \subset \mathbb{R}^n$ be a convex body. Estimate the Banach-Mazur distance from below

$$d_{BM}\Big(K, B_\infty^n\Big) \geq ?$$

The estimate should be better than the known bound $n^{\frac{5}{9}}$ which is due to Tikhomirov.

Problem 16 (Hosle). Let K, L be convex bodies, such that

$$\left|K \cap \theta^\perp\right| \leq \left|L|\theta^\perp\right|.$$

Can we then conclude that there exists a constant C_n, such that

$$|K| \leq C_n\,|L|?$$

Partial progress is known due to Johannes Hosle.

Problem 17 (Park). Let

$$V_p(K, L) = c_n \int\limits_{S^{n-1}} h_L^p d\rho_K,$$

and

$$h_{\Pi_p K} = c_n \int\limits_{S^{n-1}} |\langle x, y\rangle| dS_k.$$

Assume that K is a symmetric convex body, $|\partial K| = 1$, and $p > 0$ is not even. Is it true that

$$\min_K V_p(K, \Pi_p K)$$

is polyhedral under the above conditions?

It is known to be true for $p = 1$, due to Bilyk, Glazyrin, Matzke, Park, Vlasiuk.

Problem 18 (Roysdon). Can one prove a functional version of Dar's conjecture?

Problem 19 (Huang). Let K be an isotropic convex body, and X be a random variable uniformly distributed on K $\Big(X \sim unif\,(K)\Big)$. Let $Y = \frac{X}{|X|}$. How far is Y from $unif\,(S^{n-1})$?

Problem 20 (Livshyts).

a) Let A be an $n\times n$, $\frac{1}{2}$-Bernoulli random matrix. Consider the vector $v \perp Ae_1, \dots, Ae_{n-1}$. Find $\|v\|_p$ for $\forall p \geq 1$ (and for $p \to \infty$).

b) Read the paper of Nguyen, Vu "Normal vector of a random hyperplane" (2016).

Problem 21 (Paouris). Let f be a convex Lipschitz function on $[0,1]^n$. It is known that

$$\mathbb{P}\Big(f(X) - m_f > t\Big) \leq e^{-\frac{t^2}{c}},$$

if $X \subset [0,1]^n$ with independent coordinates, and m_f is the median. Can one prove a similar result for sub-Gaussian X? (addendum – no, a counterexample by Tikhomirov.)

Problem 22 (Paouris). Let X be a random vector in $\mathbb{R}^n$, and f be a convex function. The following estimate is known for Gaussian random vector (see a paper by Paouris, Valettas)

$$\mathbb{P}\Big(f(x) < m_f - t\Big) \leq \frac{1}{2}\exp\Big\{-\frac{t^2}{Var\,(f(x))}\Big\}.$$

Can one extend this to non-Gaussian random vector under the condition

$$k_1\, I_n \leq Hess\,(f) \leq k_2 I_n?$$

Problem 23 (Paouris). It is known that

$$\big|K \cap F^{\perp}\big| \cdot \Big|K|F\Big| \leq \binom{n}{k}|K|$$

for a subspace F, dim $F = k$. Can one replace K with Z_k, where Z_k is L_q-centroid body, and $\binom{n}{k}$ with C^k for a constant $C > 0$?

Problem 24 (Livshyts). Estimate L_K where K is a random polytope with random facets (it is known for vertices, see Klartag-Kozma). See a paper by Koldobsky, Paouris, Zvavitch in conjunction with a paper by Klartag-Koldosbky, unless $L_{K^\circ} \leq CL_K$ for other reasons.

Problem 25 (Lirola). Prove Mahler's conjecture in some special cases. In particular:

- read the paper by Karasev;
- read the paper by Artstein-Avidan and Ostrover.

2 Groups

1. Floating bodies group: Problems 1, 2, 5, 6. Shiri Artstein-Avidan (TAU), Elisabeth Werner (Case Western), Carrie Clarck (U Toronto), Ludmila Kryvonos (Kent State), Kasia Wyczesany (Cambridge), Chase Reuter (U North Dakota), Gulnar Aghabalayeva (U North Dakota), Kateryna Tatarko (University of Alberta), Alina Stancu (Concordia).

2. Radon transform in convexity group: Problems 3, 4. Alexander Koldobsky (U Missouri), Michael Roysdon (Kent State), Sudan Xing (University of Alberta), Shay Sadovsky (TAU), Katharina Eller (Tech U Berlin).

3. Sections and projections group: Problems 10, 16, 23. Mariangel Alfonseca (U North Dakota), Ryan Matzke (U Minnesota), Luis C. Garcia-Lirola (Kent State), Jesus Rebollo Bueno (U Missouri), Susanna Dann (U Chilie), Giorgis Chaspis (Kent State).

4. Ellipsoids and slicing group: Problems 9, 11. Uri Grupel (U Innsbruk), Kasia Wyczesany (Cambridge), Andre Wisibono (Georgia Tech), Santosh Vempala (Georgia Tech), Yin Tat Lee (Washington U), Ryan Gibara (Concordia), Ansgar Freyer (Tech U Berlin).

5. Delocalization group: Problems 19, 20. Sergey Myroshnychenko (U Alberta), Gautam Aishwarya (U Delaware), Irfan Alam (Louisiana State), Oscar Zatarain-Vera (Kent State), Dongbin Li (U Delaware). Brief appearance by Konstantin Tikhomirov (Georgia Tech).

6. Concentration group: Problems 21, 22, 12. Xingyu Zhu (Georgia Tech), Han Huang (Georgia Tech), Galyna Livshyts (Georgia Tech), Max Fathi (Toulouse), Grigoris Paouris (Texas A&M), Alexander Kolesnikov (Higher School Econ.), Shay Sadovsky (TAU).

7. Harmonic Analysis and discrepancy group: Problem 17. Josiah Park (Georgia Tech), Benjamin Jaye (Clemson U), Sudan Xing (U Alberta).

8. Banach-Mazur distance group: Problem 15. Johannes Hosle (UCLA), Konstantin Tikhomirov (Georgia Tech), Alexander Litvak (U Alberta), Michail Sarantis (Georgia Tech).

9. Random matrices group: Problems 13, 14. Anna Lytova (U Opole), Josiah Park (Georgia Tech). Brief appearance by Konstantin Tikhomirov (Georgia Tech).

Figure 1: Group 1: Floating bodies. Currently present Ludmila Kryvonos, Chase Reuter, Carrie Clarck, Gulnar Aghabalayeva. Currently absent Shiri Artstein-Avidan, Elisabeth Werner, Kateryna Tatarko.

Figure 2: Group 2: Radon transform in convexity. Currently present Sudan Xing, Michael Roysdon.

Figure 3: Group 3: Sections and projections. Currently present Mariangel Alfonseca, Ryan Matzke, Luis C. Garcia-Lirola, Jesus Rebollo Bueno, Giorgis Chaspis.

Figure 4: Group 4: Ellipsoids and slicing. Currently present Ryan Gibara, Uri Grupel, Ansgar Freyer. Currently absent Santosh Vempala, Andre Wisibono, Kasia Wyscez-nanny, Yin Tat Lee.

Figure 5: Group 5: Delocalization. Sergey Myroshnychenko, Gautam Aishwarya, Irfan Alam, Oscar Zatarain-Vera, Dongbin Li.

Figure 6: Group 6: Concentration. Currently present Grigoris Paouris, Han Huang, Alexander Kolesnikov, Max Fathi; currently absent Zhu, Livshyts.

Figure 7: Group 8: Banach Mazur distance to the cube. Alexander Litvak, John Hosle, Konstantin Tikhomirov.

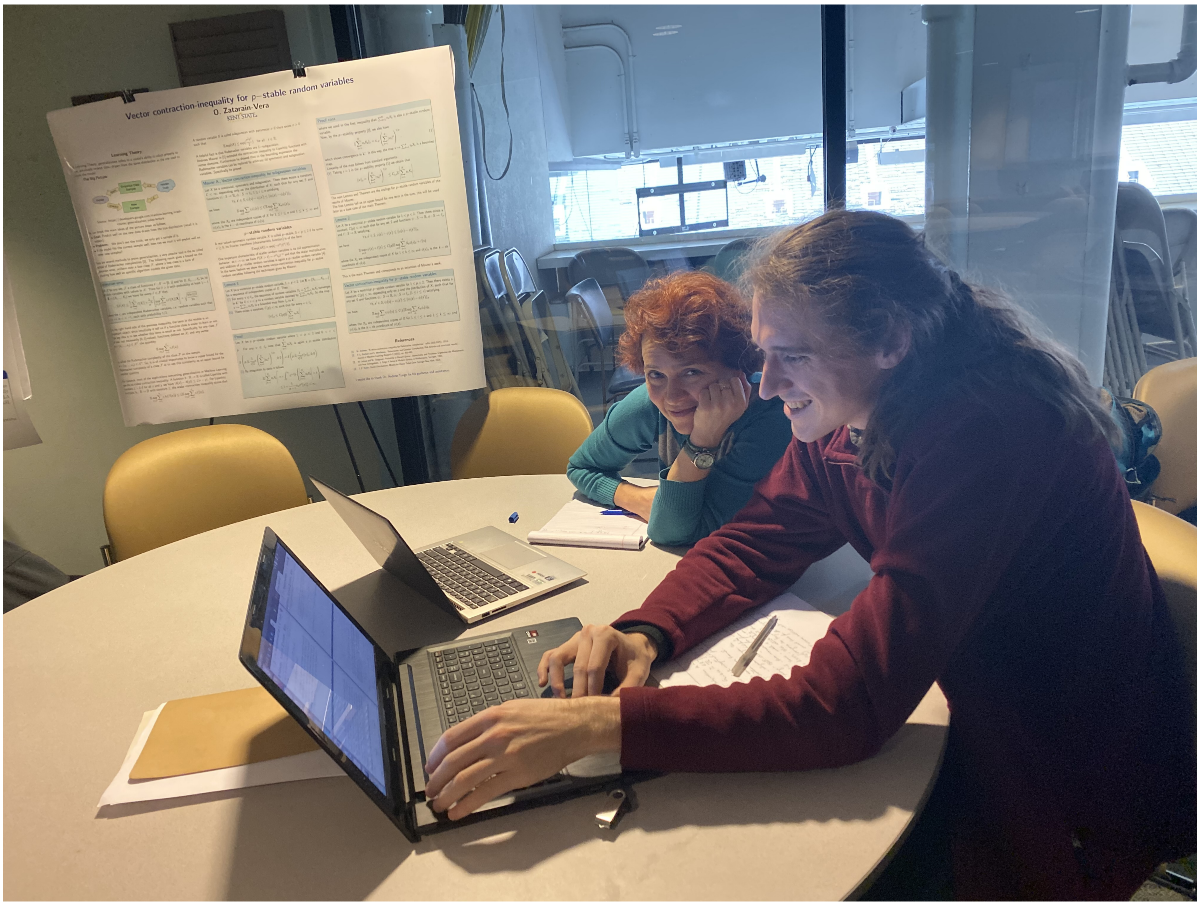

Figure 8: Group 9: Random Matrices. Anna Lytova, Josiah Park.

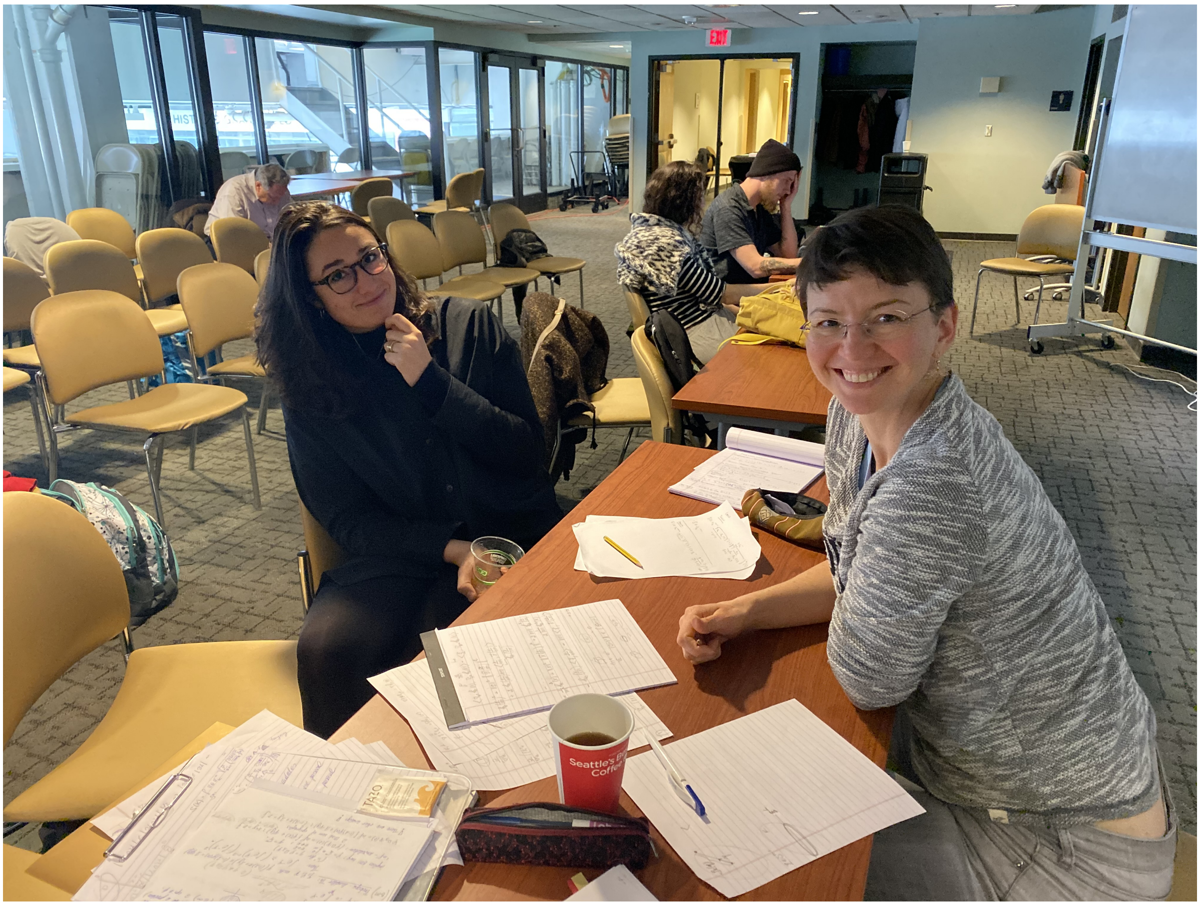

Figure 9: Discussions for Group 2 by Katharina Eller, Susanna Dann.

Figure 10: Discussions for Group 2 by Shay Sadovsky, Michael Roysdon.

Figure 11: Additional discussions during lunch by Shay Sadovsky, Shiri Artstein-Avidan, Kasia Wysceznanny.

Figure 12: An efficient use of a double-sided whiteboard by Alexander Kolesnikov, John Hosle, Josiah Park, Grigoris Paouris.

Figure 13: Additional discussions by Susanna Dann, Katryna Tatarko, Anna Lytova.

Figure 14: Additional discussions by Katryna Tatarko, Ludmyla Kryvonos.

Figure 15: Additional discussions Group 3 by Maria Alfonseca, Dima Ryabogin.

Figure 16: Additional discussions Group 6 by Han Huang, Grigoris Paouris, Alexander Kolesnikov, Max Fathi.

Figure 17: Additional discussions Group 1 by Chase Reuter, Carrie Clarck, Gulnar Aghabalayeva.